Numerical simulation of small-scale mixing processes in the upper ocean and atmospheric boundary layer

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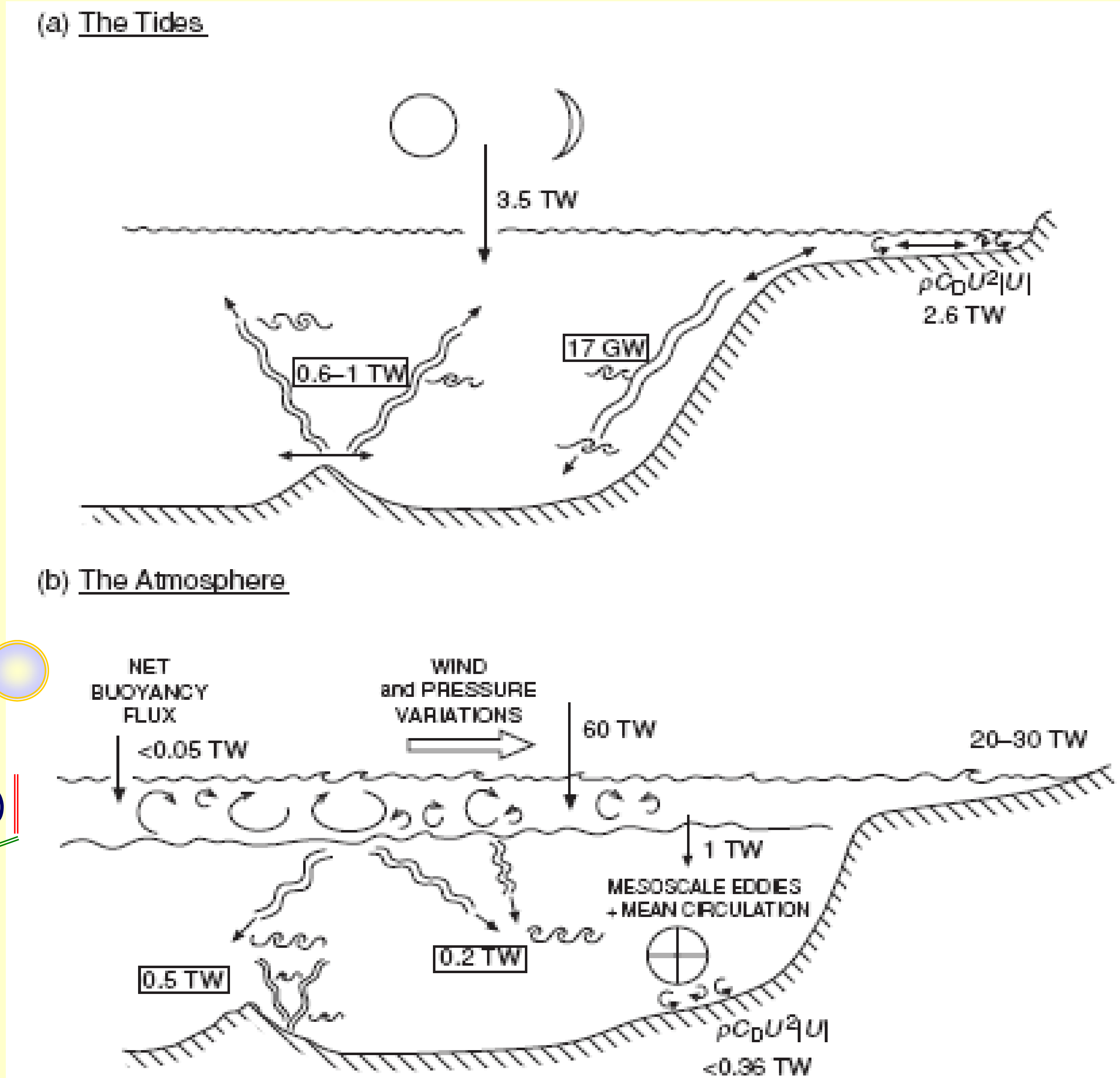
Mixing in the upper ocean and atmospheric boundary layer



O(10m)

Thorpe (2007)

Numerical approaches:

- **RANS** (Reynolds-averaged Navier-Stokes Equations): considers statistically-stationary mean flow fields and models fluxes due to small-scale (turbulent) motions as functions of the gradients of mean fields. Typical scales - O(1km-10km)

- **LES** (Large-Eddy Simulation): in addition to predicting mean flow fields resolves also large-scale, energetically dominating components of the turbulent fields. Models subgrid turbulent motions as functions of the gradients of the large-scale fields. Typical scales – O(10m-100m);

- **DNS** (Direct Numerical Simulation): solves the full, 3D Navier-Stokes equations without any modeling and resolves all relevant flow field components. Typical scales – O(10cm – 1m).

OBJECTIVE

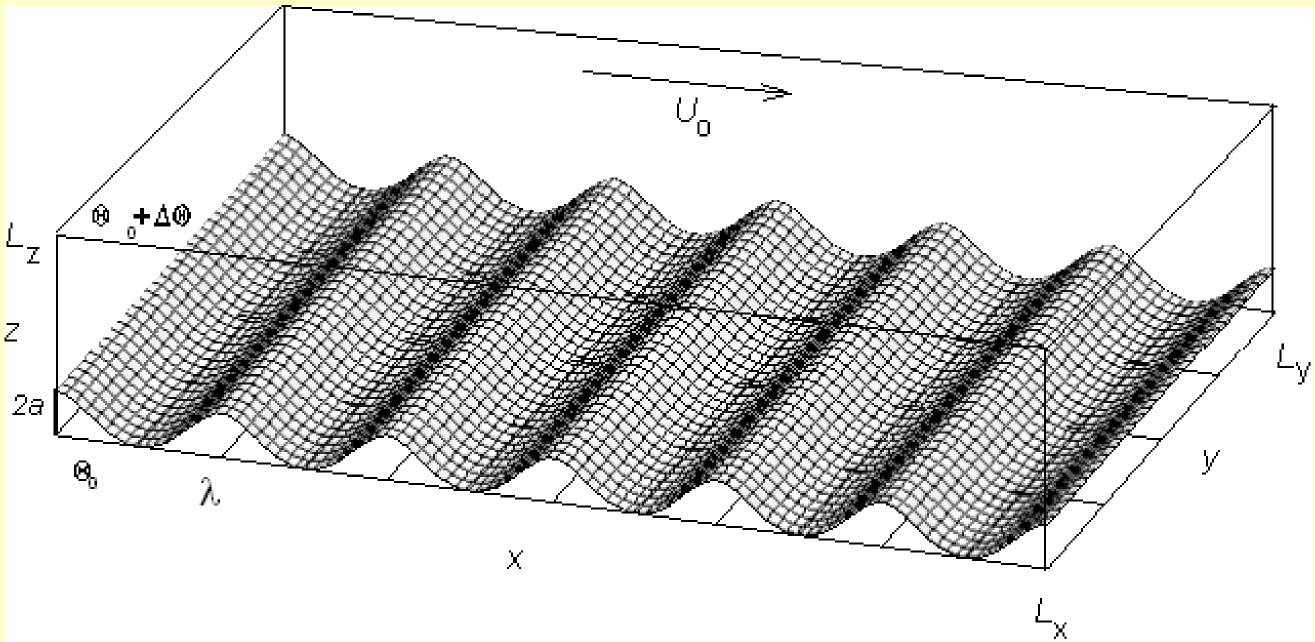
•The processes of turbulent mixing and momentum and heat exchange occur in the upper ocean at depths up to several dozens of meters and in the atmospheric boundary layer within interval of millimeters to dozens of meters and can not be resolved by known large-scale climate models.

•Thus small scale processes need to be parameterized with respect to large scale fields. These parameterizations relate turbulent fluxes with large-scale fields gradients and are dependent on the properties of the small-scale mixing processes. These dependencies are not well understood at present and need to be clarified.

•We employ Direct Numerical Simulation (DNS) as a research tool which resolves all relevant flow scales, and does not require closure assumptions typical of Large-Eddy and Reynolds Averaged Navier-Stokes simulations (LES and RANS).

• Here we discuss the problems of the interaction between small-scale turbulence and internal waves in the upper ocean as well as the effect of the surface waves on the atmospheric boundary layer over water surface.

Numerical simulation of atmospheric boundary layer over wavy water surface



$$L_x = 6\lambda \qquad L_y = 4\lambda \qquad L_z = \lambda$$

GOVERNING EQUATIONS:

The Navier-Stokes eqs. under the Boussinesq approximation

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{\partial P}{\partial x_j} + \frac{1}{\mathrm{Re}} \frac{\partial^2 U_i}{\partial x_j \partial x_j} + \delta_{iz} Ri \widetilde{T} f(t)$$

$$\frac{\partial U_j}{\partial x_j} = 0.$$

$$\frac{\partial \widetilde{T}}{\partial t} + \frac{\partial (U_j \widetilde{T})}{\partial x_j} + U_z = \frac{1}{\mathrm{Pr}\mathrm{Re}} \frac{\partial^2 \widetilde{T}}{\partial x_j \partial x_j}$$

$$\mathrm{Re} = \frac{U_0 \lambda}{\nu} \qquad \mathrm{Ri} = g \frac{T_2 - T_1}{T_1} \frac{\lambda}{U_0^2}$$

$$T = 1 + z + \widetilde{T} \qquad f(t) = 1 - \exp(-t/100)$$

WAVE-FOLLOWING CURVILINEAR COORDINATES

$$x = \xi - a\exp(-k\eta)\sin k\xi$$

$$z = \eta + a\exp(-k\eta)\cos k\xi$$

$$z_b(x) = a\cos kx + \frac{1}{2}a^2k(\cos 2kx - 1)$$

Mapping over η: $\eta = 0.5\left(1 + \frac{\tanh\tilde{\eta}}{\tanh 1.5}\right)$ $-1.5 < \tilde{\eta} < 1.5$

BOUNDARY CONDITIONS

Bottom plane:

$$U(\xi, y, 0) = c\left(ka \cos kx(\xi, \eta) - 1\right)$$

$$V(\xi, y, 0) = 0$$

$$W(\xi, y, 0) = cka \sin kx(\xi, \eta)$$

Top plane:

$$U(\xi, y, 1) = 1 - c$$

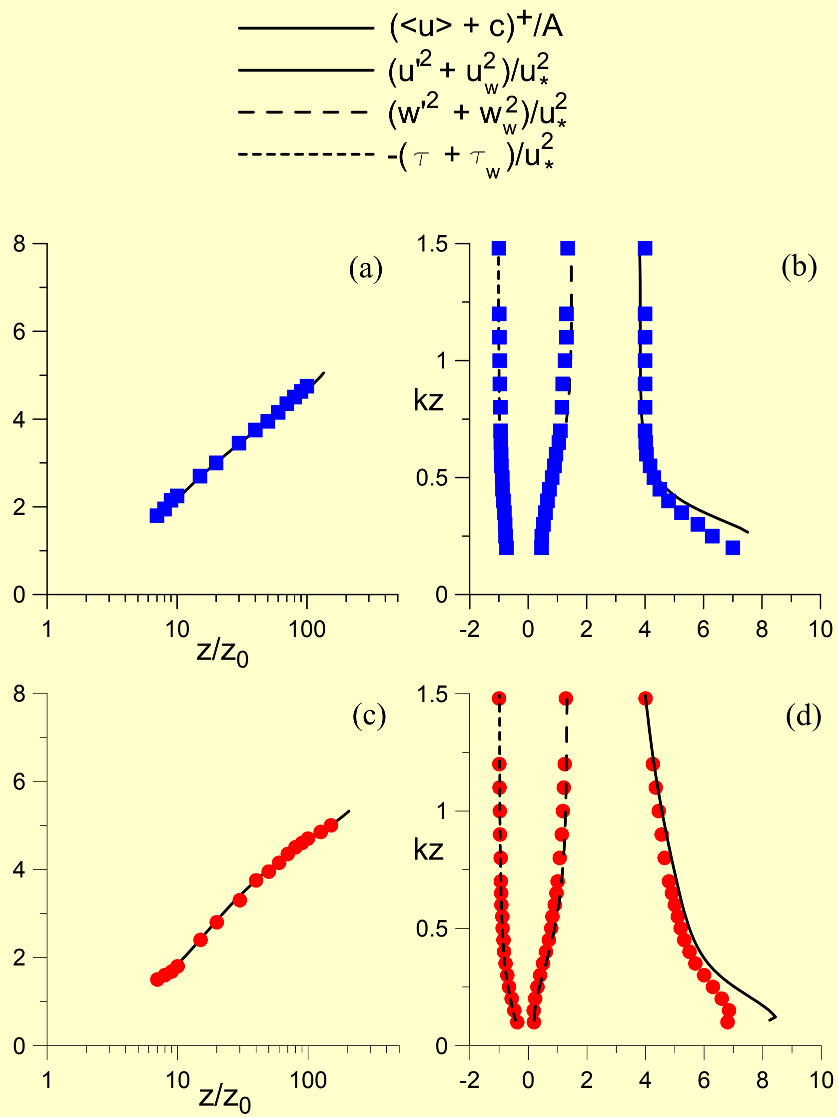
$$V(\xi, y, 1) = 0$$

$$W(\xi, y, 1) = 0$$

$$\widetilde{T}(\xi, y, 0) = \widetilde{T}(\xi, y, 1) = 0$$

All fields are x and y periodic

CODE VALIDATION: COMPARISON WITH DNS BY SULLIVAN ET AL. (2000)

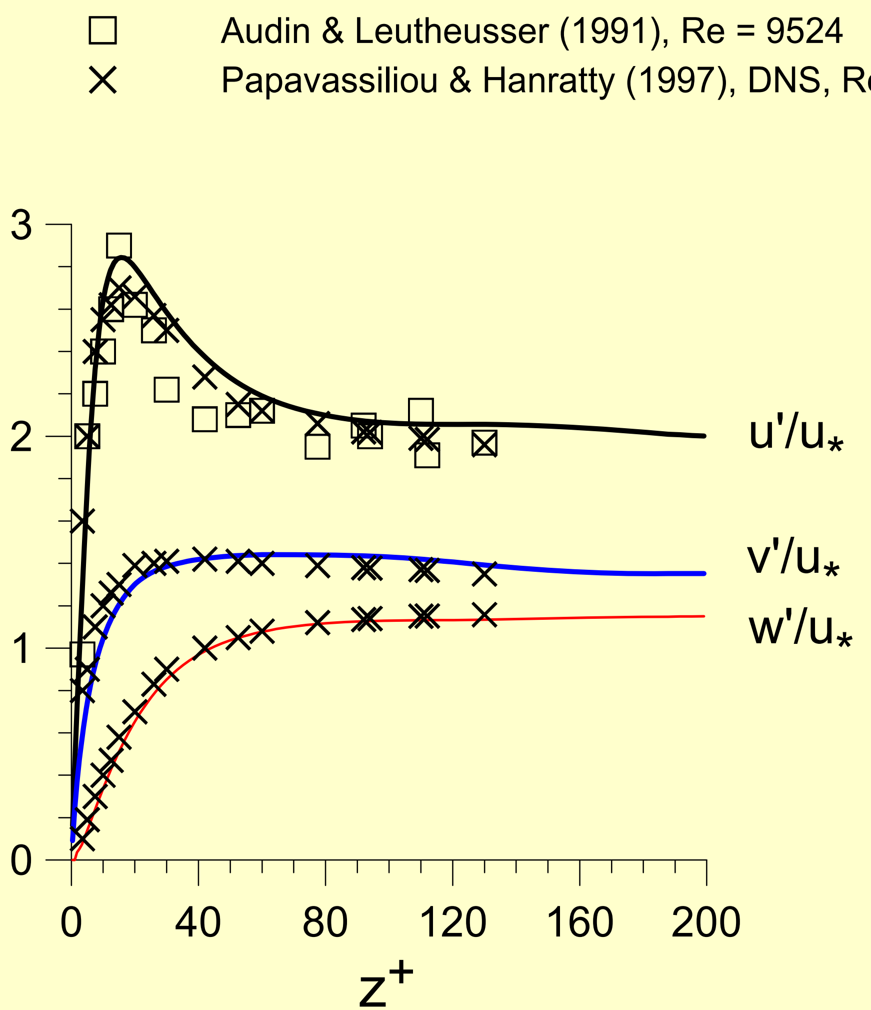


Re = 10000, c=0.25

ka = 0.2

ka = 0.1

CODE VALIDATION: COMPARISON WITH LABORATORY-EXPERIMENT RESULTS

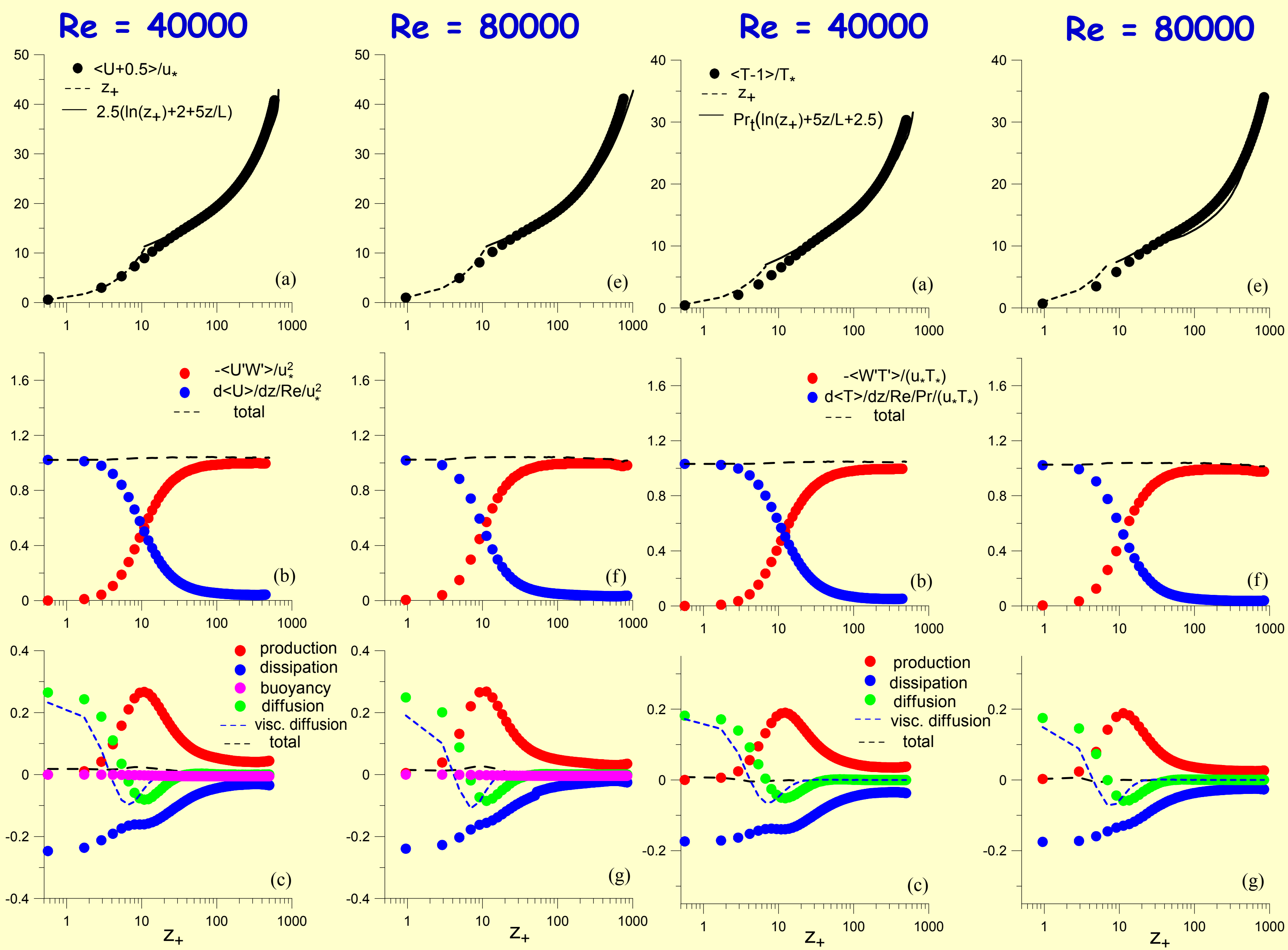


CODE VALIDATION: COMPARISON WITH MONIN-OBUKHOV THEORY

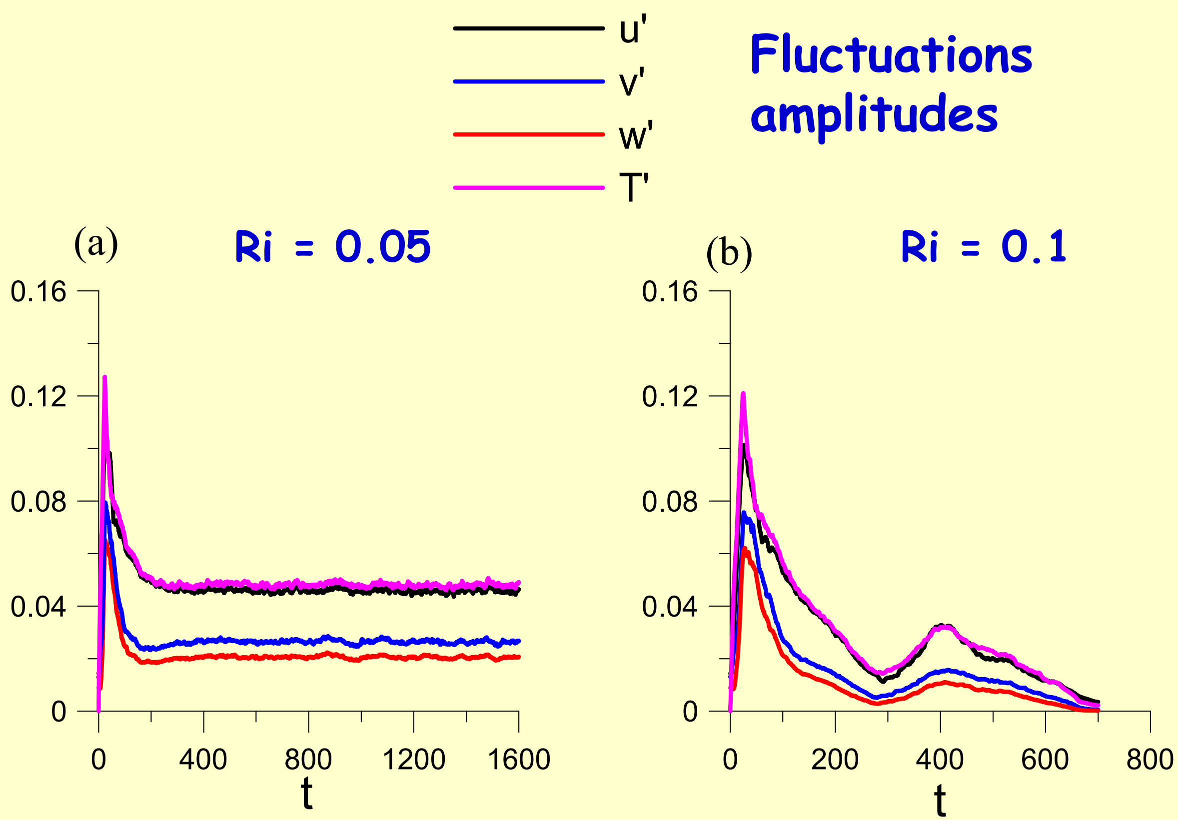
Mean velocity and temperature profiles

Momentum and heat flux budget

Kinetic and potential energy budget



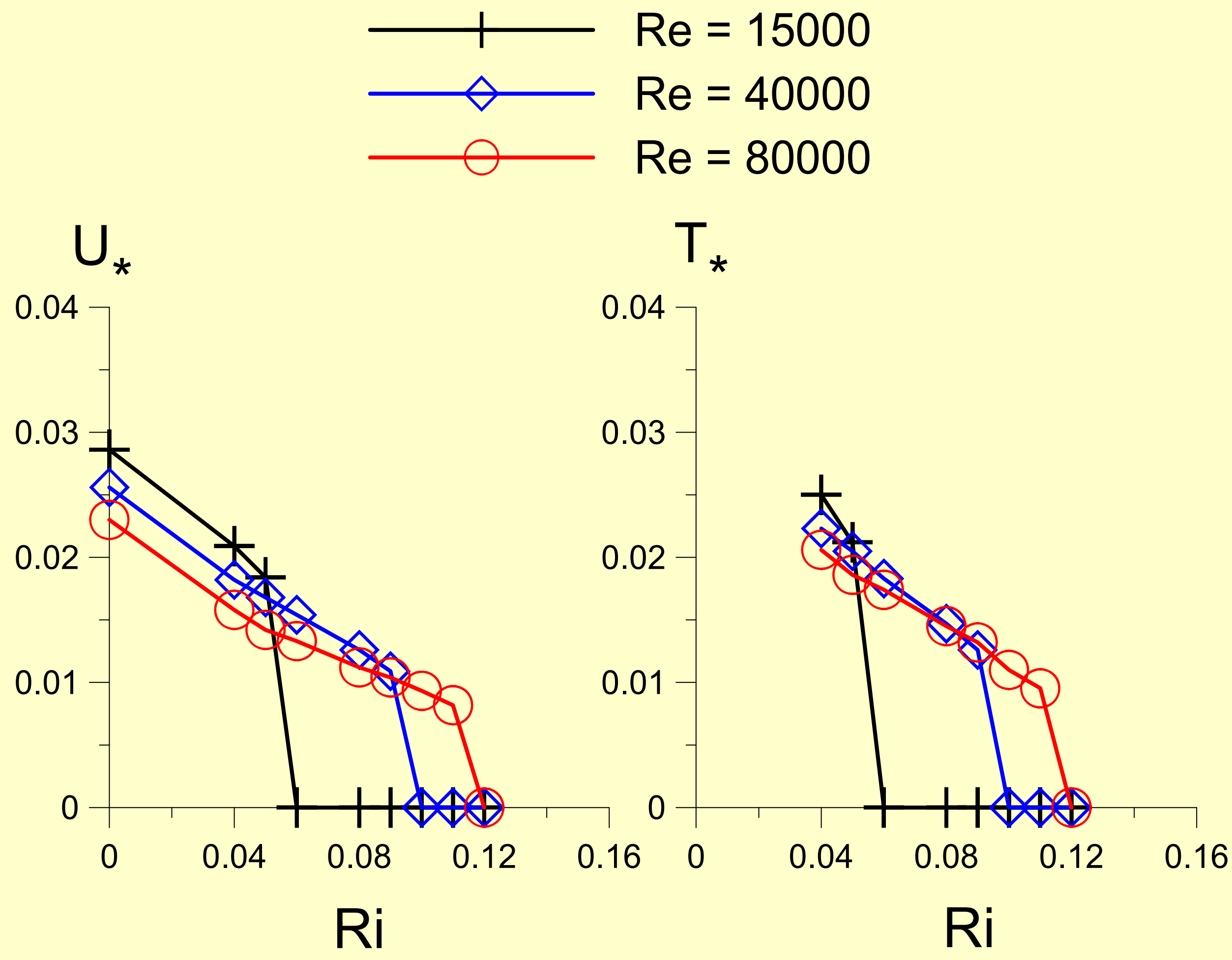
RELAMINARIZATION OF STRATIFIED FLOW ABOVE THE FLAT BOUNDARY



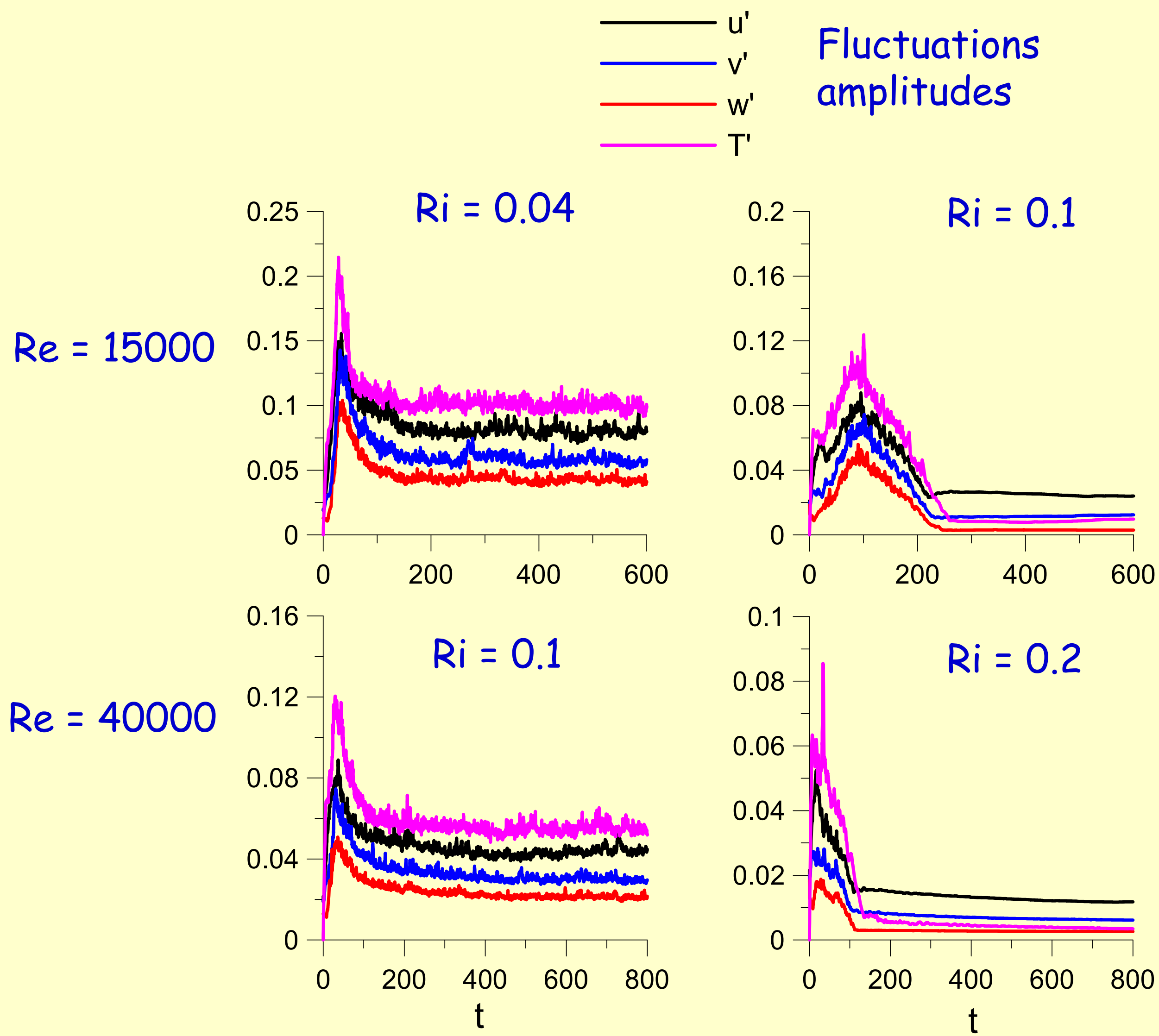
Turbulent regime

Laminar regime

RELAMINARIZATION OF STRATIFIED FLOW ABOVE THE FLAT BOUNDARY

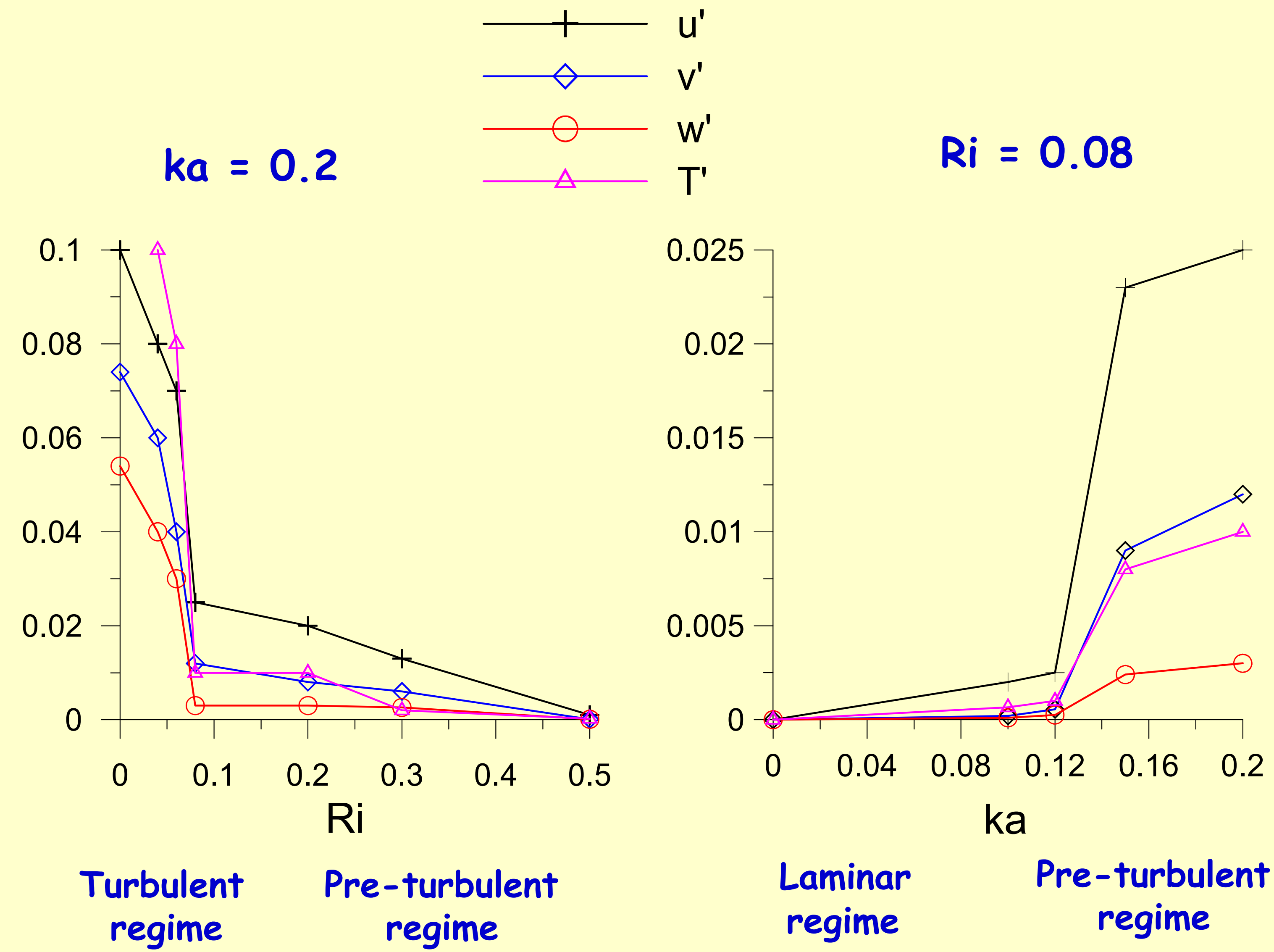


EFFECT OF THE WAVE
(ka = 0.2, c = 0.05)



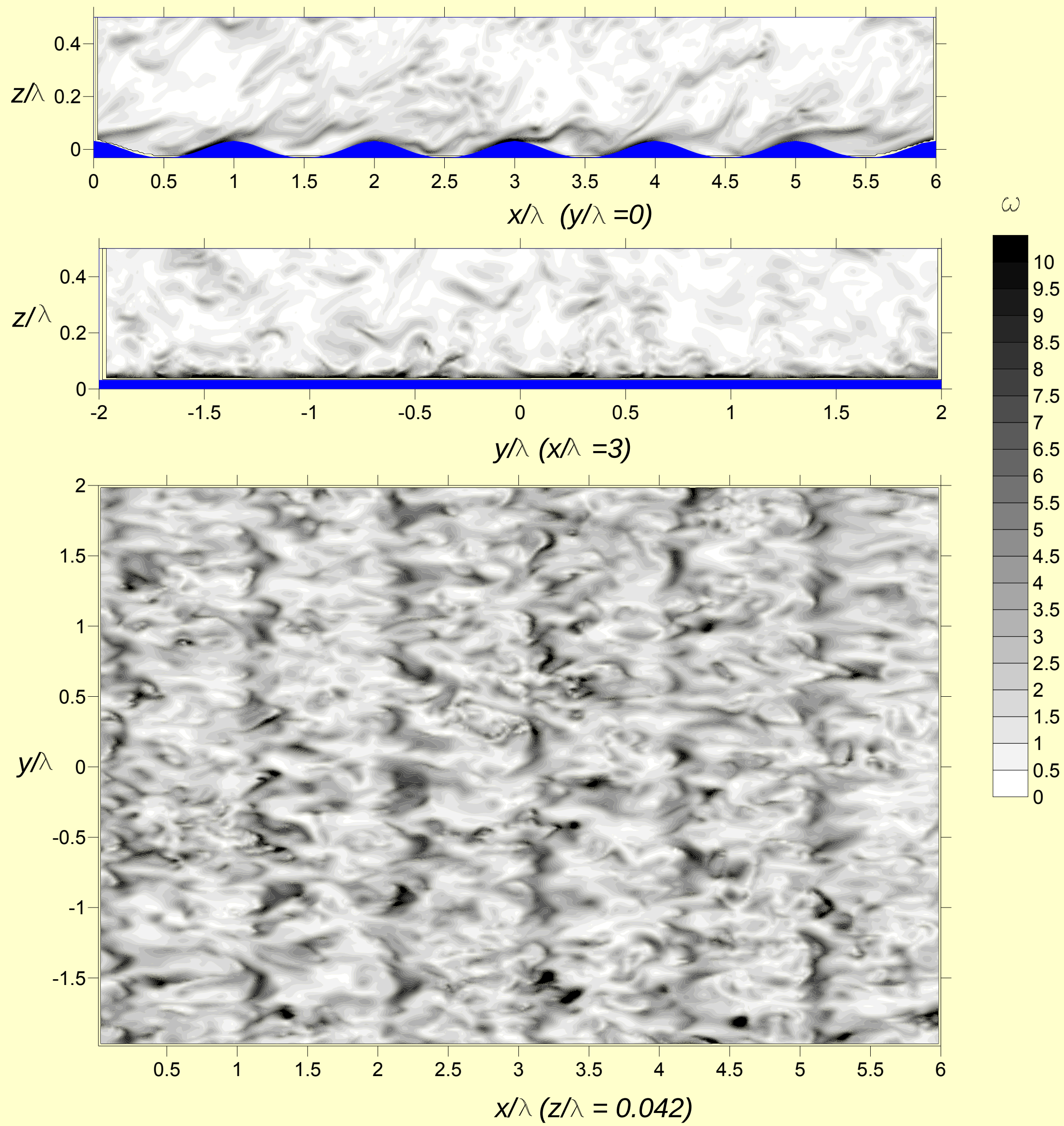
Turbulent regime **Pre-turbulent regime**

EFFECT OF THE WAVE

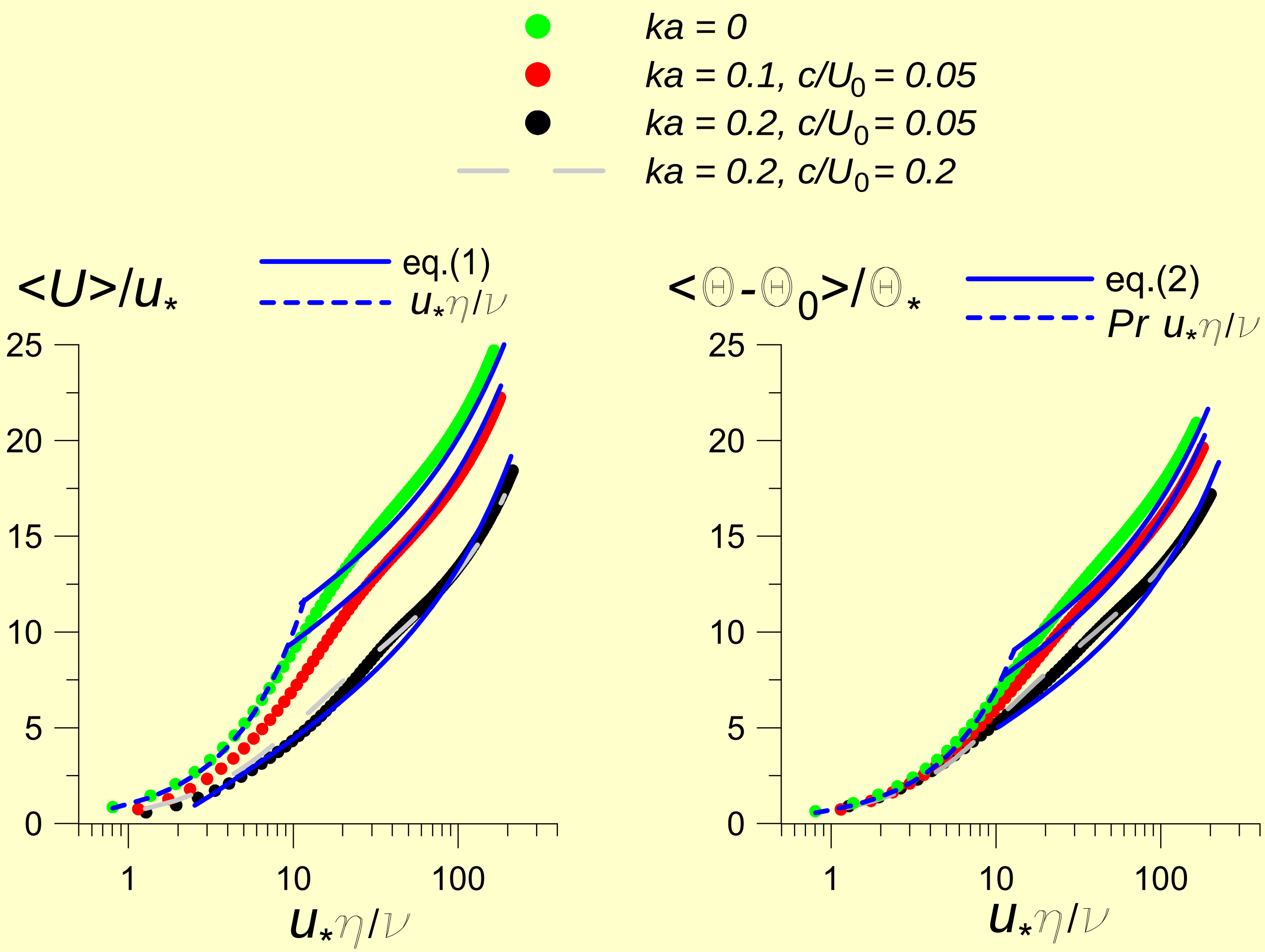


INSTANTANEOUS VORTICITY MODULUS FIELD: TURBULENT REGIME

(Ri = 0.05, Re=15000, ka = 0.2, c = 0.05)



TURBULENT REGIME:
Mean velocity and temperature profiles



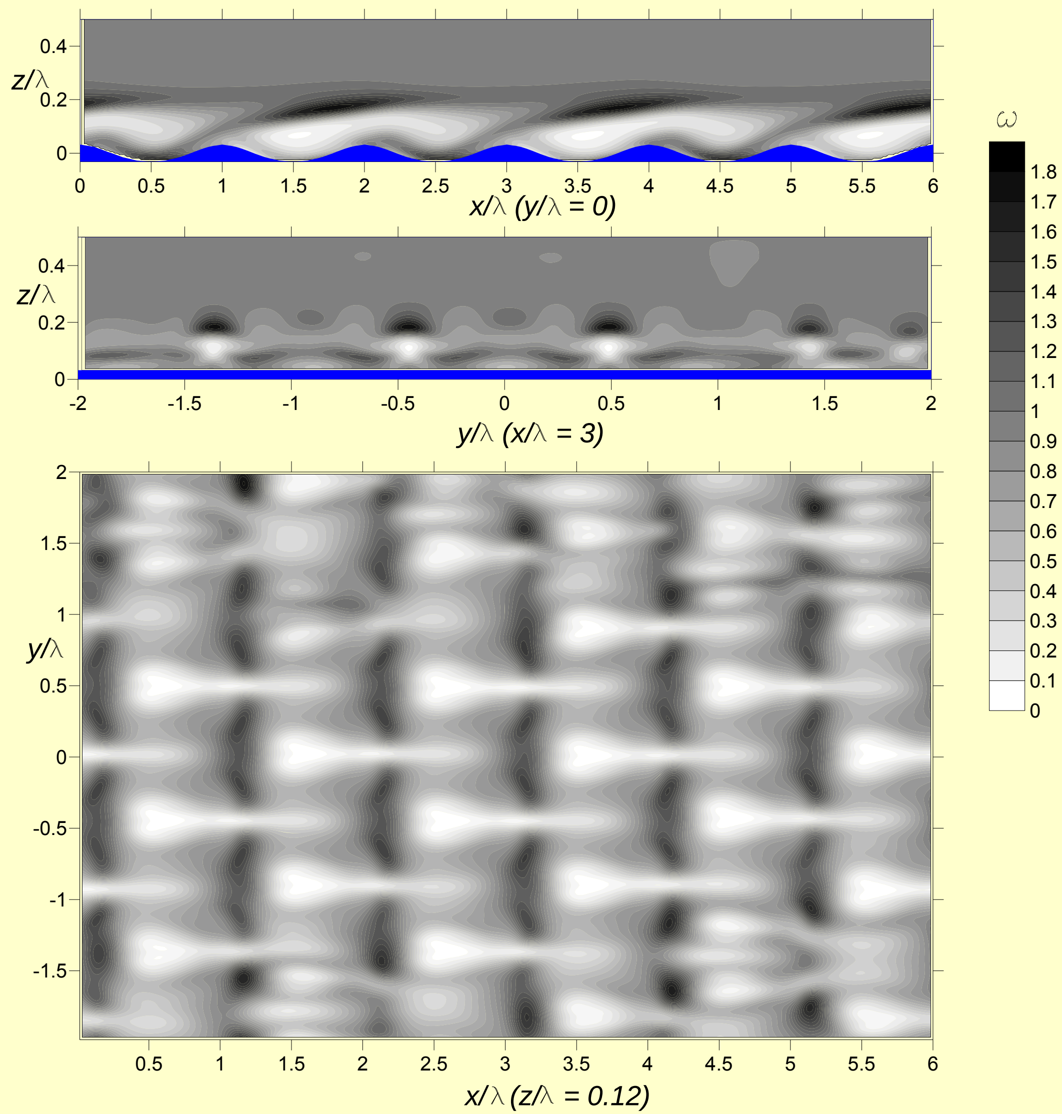
Monin-Obukhov asymptotics

$$\frac{U(\eta)}{u_*} = \frac{1}{\kappa}\left(\ln\frac{\eta}{z_{0U}} + C_U\frac{\eta}{L}\right) \quad (1)$$

$$\frac{\Theta(\eta) - \Theta_0}{\Theta_*} = \frac{\Pr_t}{\kappa}\left(\ln\frac{\eta}{z_{0\Theta}} + C_U\frac{\eta}{L}\right) \quad (2)$$

$C_U = 5$

PRE-TURBULENT REGIME:
INSTANTANEOUS VORTICITY MODULUS FIELD
(RE=15000, Ri=0.08, ka=0.2)

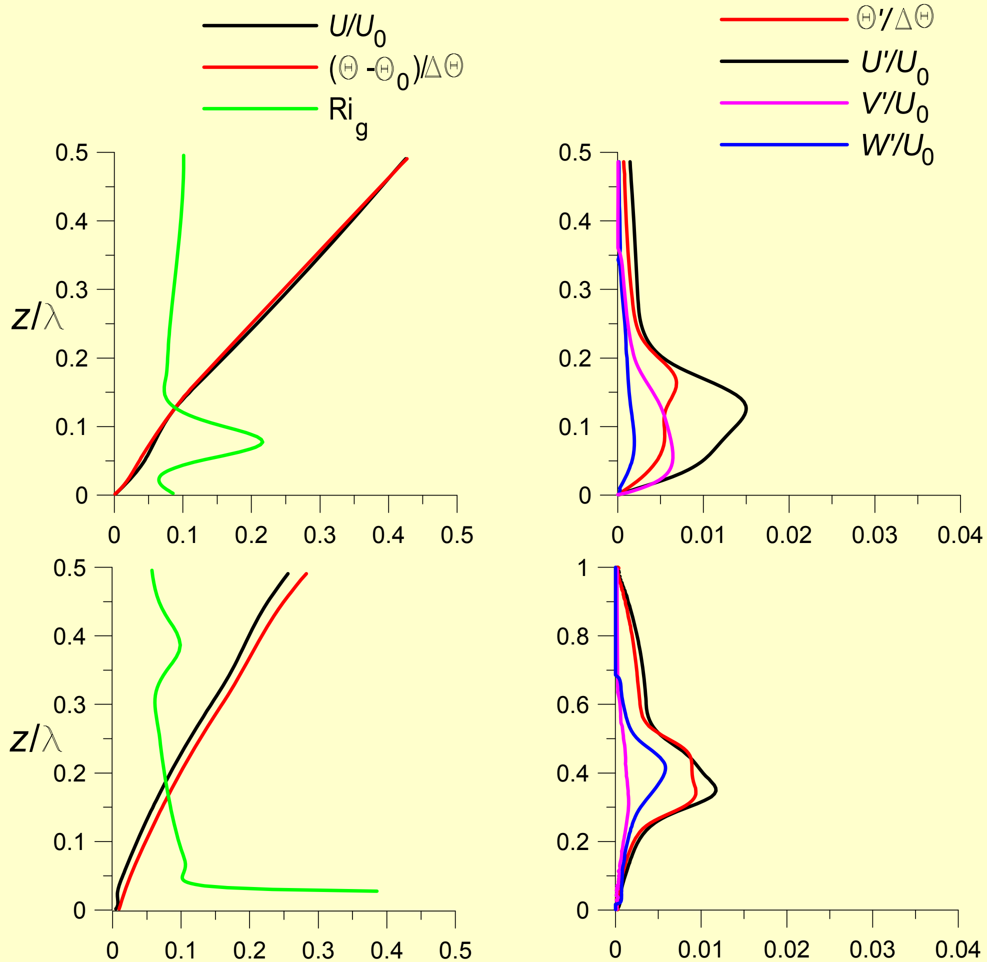


PRE-TURBULENT REGIME
(ka = 0.2, c = 0.05, Ri = 0.08)

$$Ri_g = \frac{g}{\Theta_0} \frac{d\Theta / dz}{(dU / dz)^2}$$

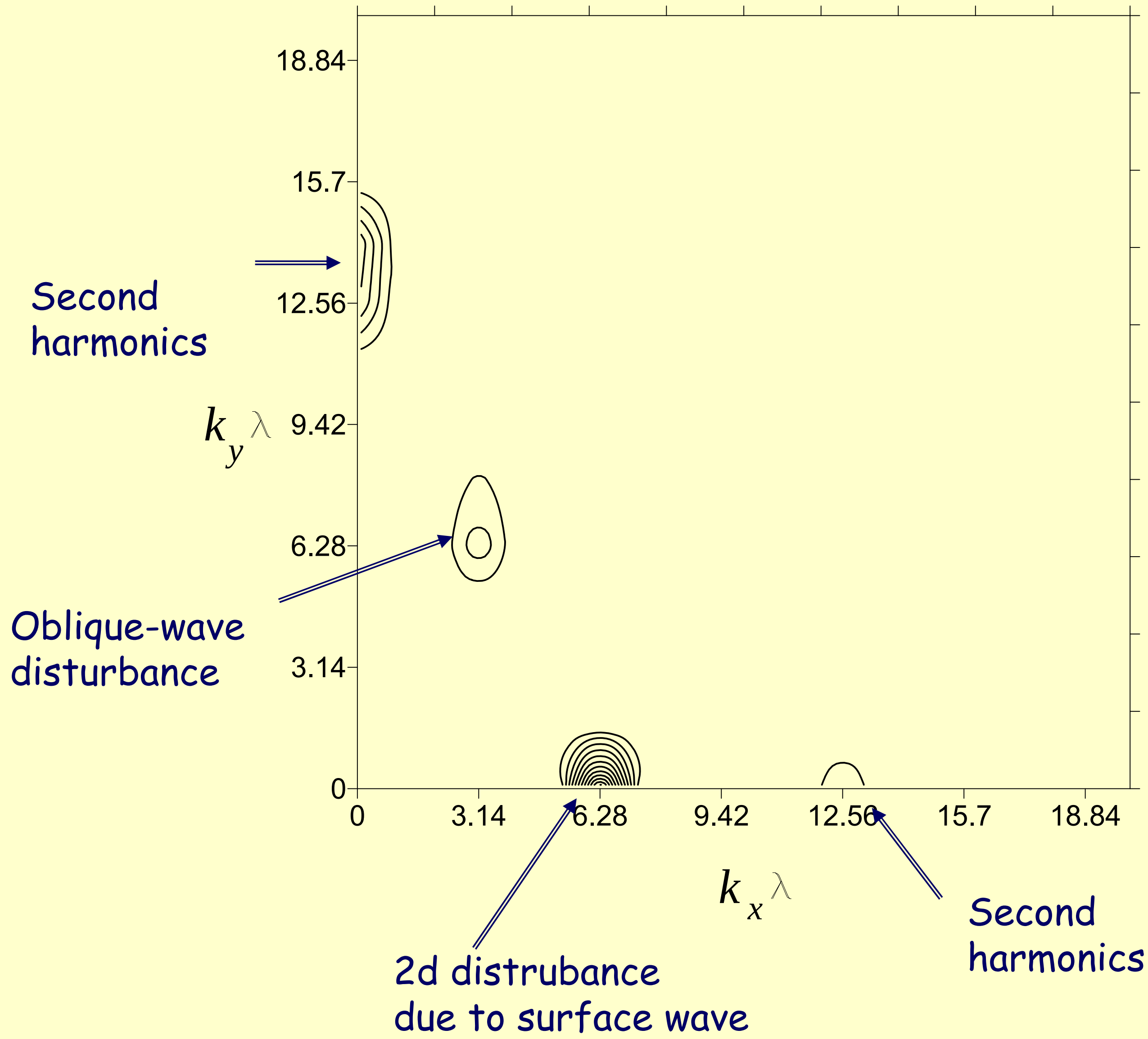
Mean velocity and temperature, and Ri_g profiles

Fluctuations profiles



PRE-TURBULENT REGIME:

POWER SPECTRUM OF THE VORTICITY FIELD



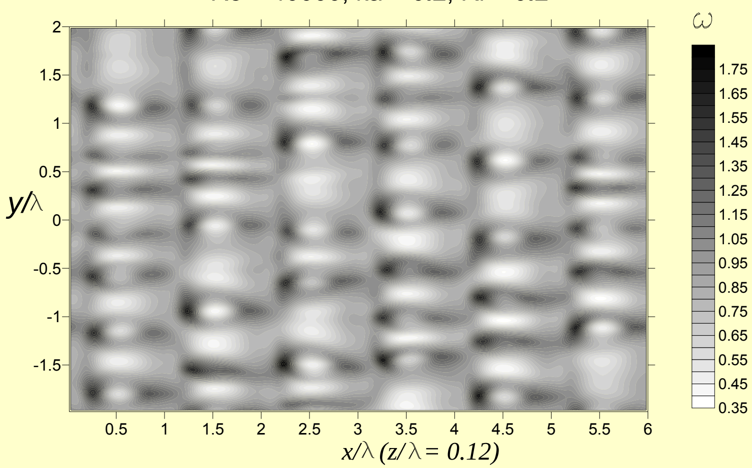
Parametric resonance condition:

$$k_{1x} = \frac{1}{2} k$$

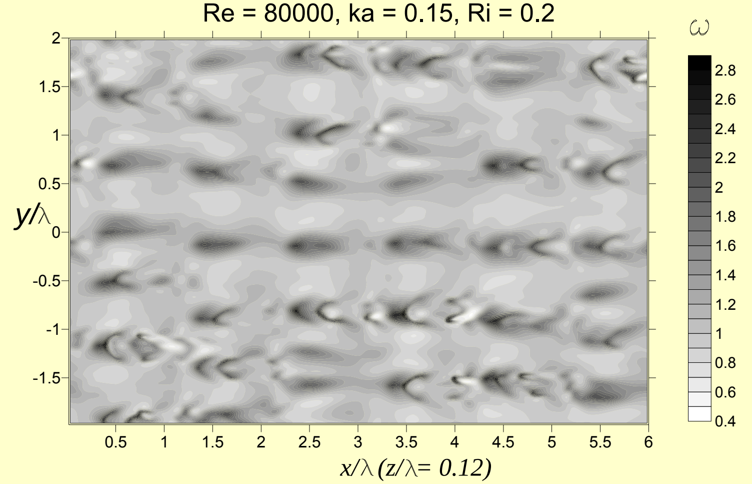
PRE-TURBULENT REGIME:
EFFECT OF INCREASING RE

Instantaneous vorticity

Re = 40000, ka = 0.2, Ri = 0.2

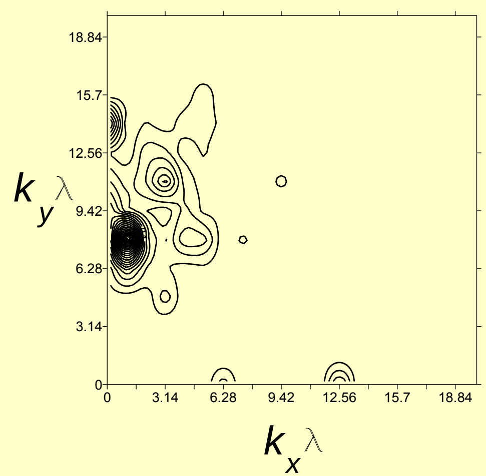


Re = 80000, ka = 0.15, Ri = 0.2

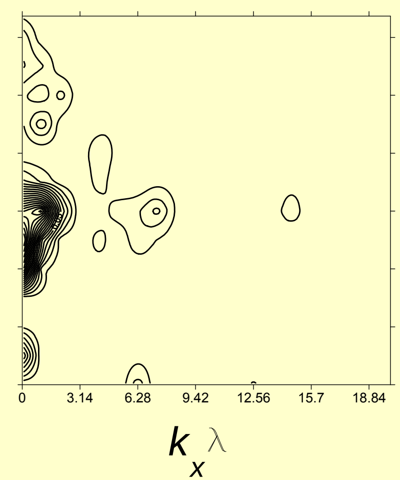


Power spectrum of the vorticity

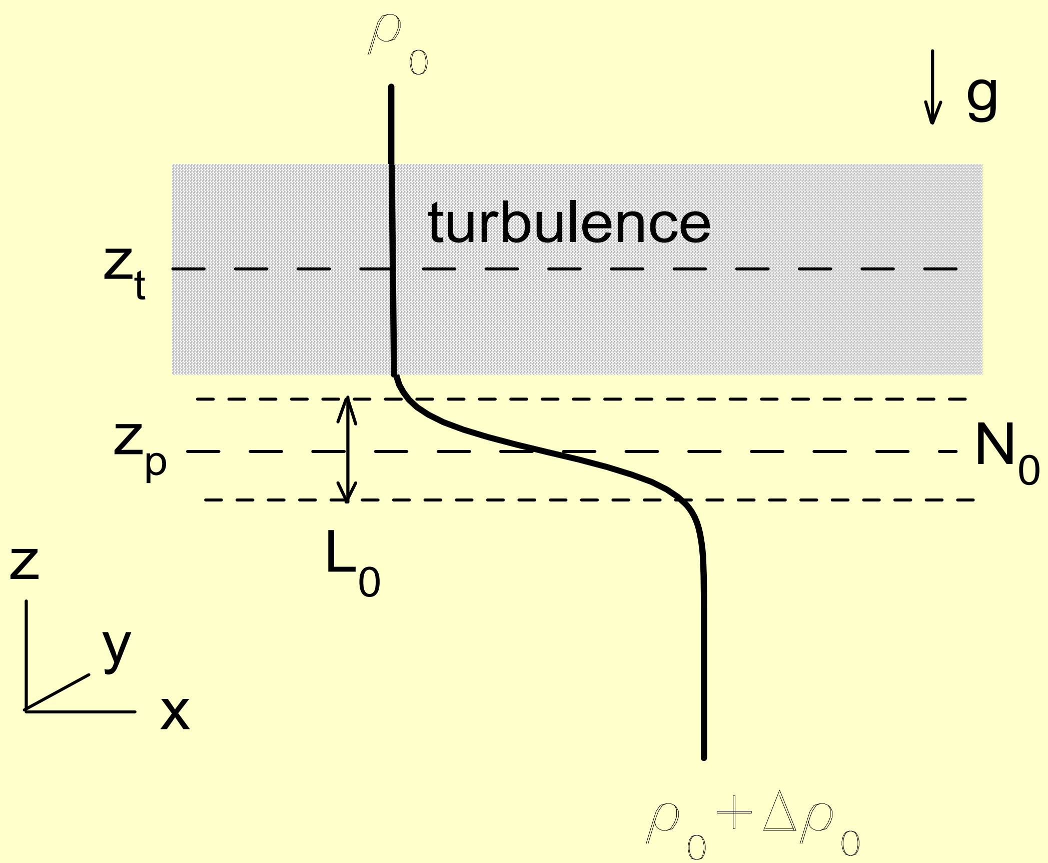
Re = 40000, ka = 0.2, Ri = 0.2



Re = 80000, ka = 0.15, Ri = 0.2



Numerical simulation of turbulence-IW interactrion



Time and velocity scales in DNS are based on the buoyancy frequency N_0 and pycnocline thickness L_0:

$$T_0 = 1/N_0 \qquad U_0 = L_0/T_0 = L_0 N_0$$

Governing equations and DNS parameters

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial^2 U_i}{\partial x_j^2} - Ri\,\delta_{iz}\rho$$

$$\frac{\partial U_j}{\partial x_j} = 0$$

$$\frac{\partial \rho}{\partial t} + U_j \frac{\partial \rho}{\partial x_j} - U_z N_0^2(z) = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \frac{\partial^2 \rho}{\partial x_j^2}$$

Reference buoyancy frequency profile: $N_0(z) = \dfrac{1}{\cosh 2(z - z_p)}$

$$\mathrm{Re} = \frac{U_0 L_0}{\nu} = 20000$$

$$Ri = \left(\frac{L_0 N_0}{U_0}\right)^2 = 1$$

Pr = 1

Scales in lab:
L_0=0.2m; U_0= 10 cm/s
N_0= 0.5 rad/s
$\lambda_{IW} \approx O(1\text{m})$;
$T_{IW} \approx O(10\text{s})$

Scales in the ocean:
L_0=20m; U_0= 10 cm/s
N_0= 0.01 rad/s
$\lambda_{IW} \approx O(100\text{m})$;
$T_{IW} \approx O(100\text{s})$

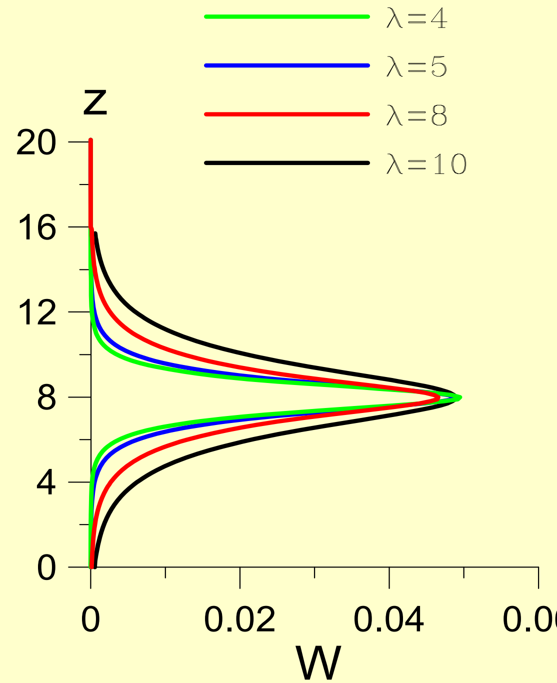
Internal Waves: initialization and characteristics

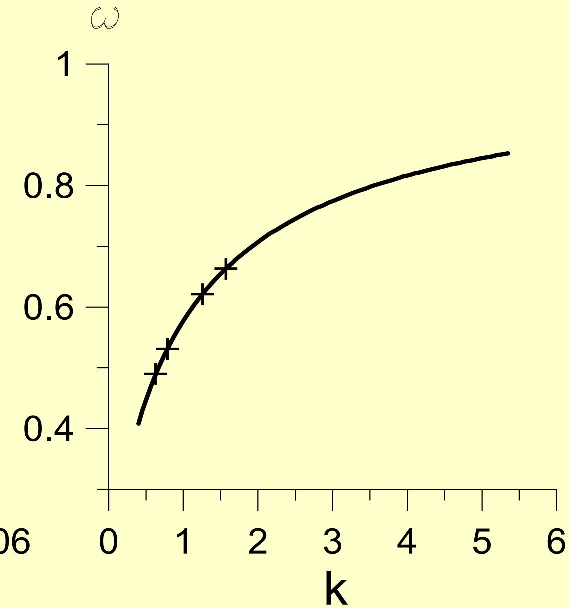
$$U_x^{IW}(x,z,t) = -\frac{1}{k}\frac{dW(z)}{dz}\sin(kx)$$

$$U_z^{IW}(x,z,t) = W(z)\cos(kx)$$

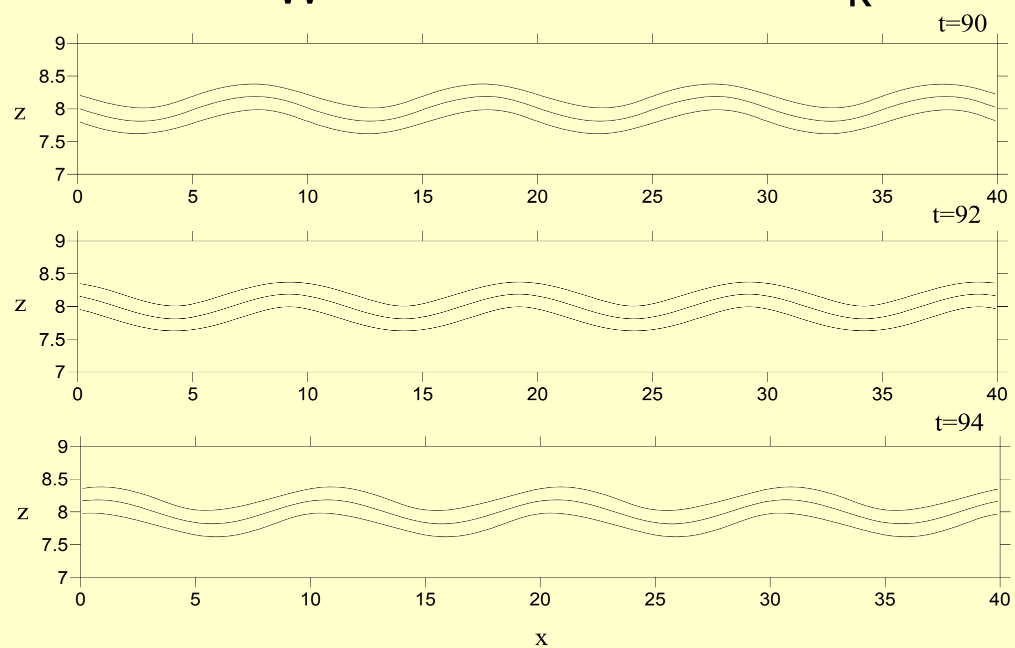
$$\rho_z^{IW}(x,z,t) = \frac{W(z)}{\omega}\frac{d\rho_{ref}}{dz}\sin(kx)$$

$$\frac{d^2W}{dz^2} + \left(\frac{N^2}{\omega^2} - 1\right)k^2W = 0$$

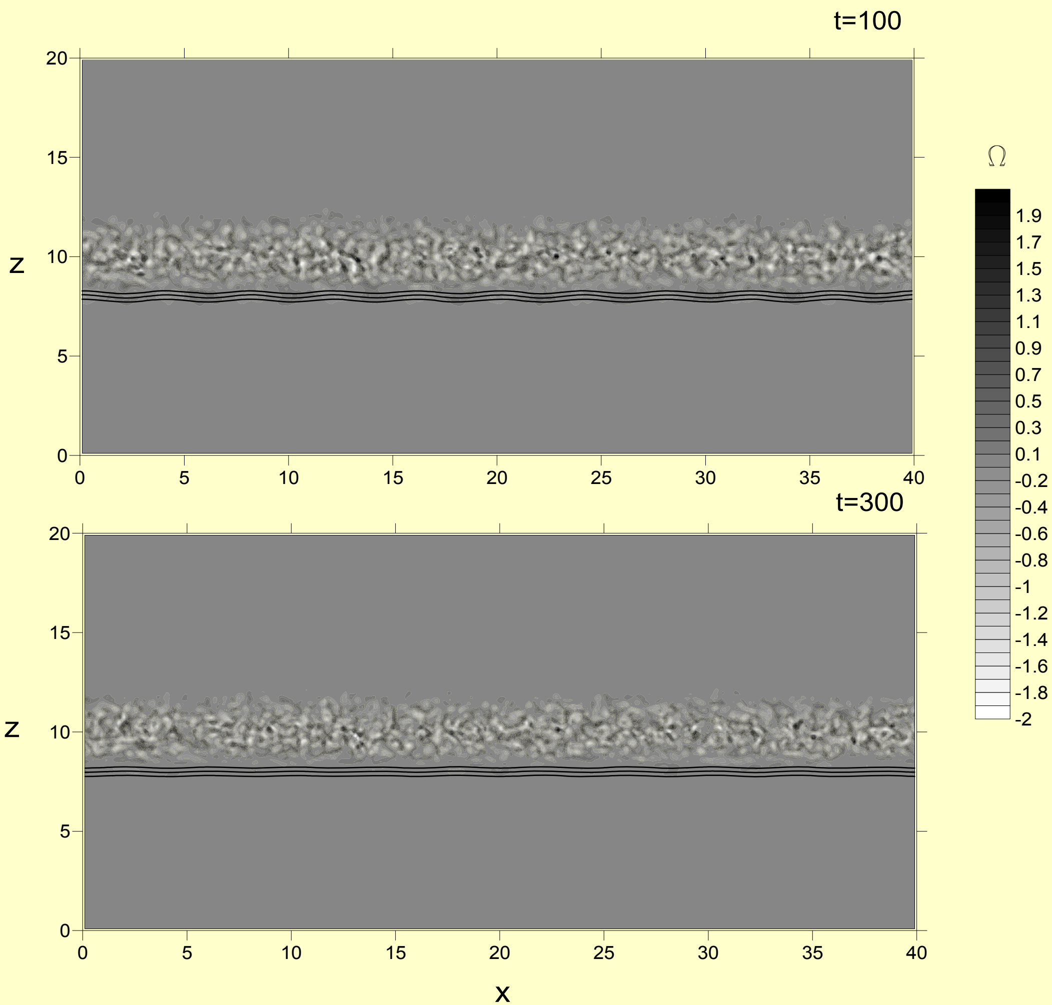


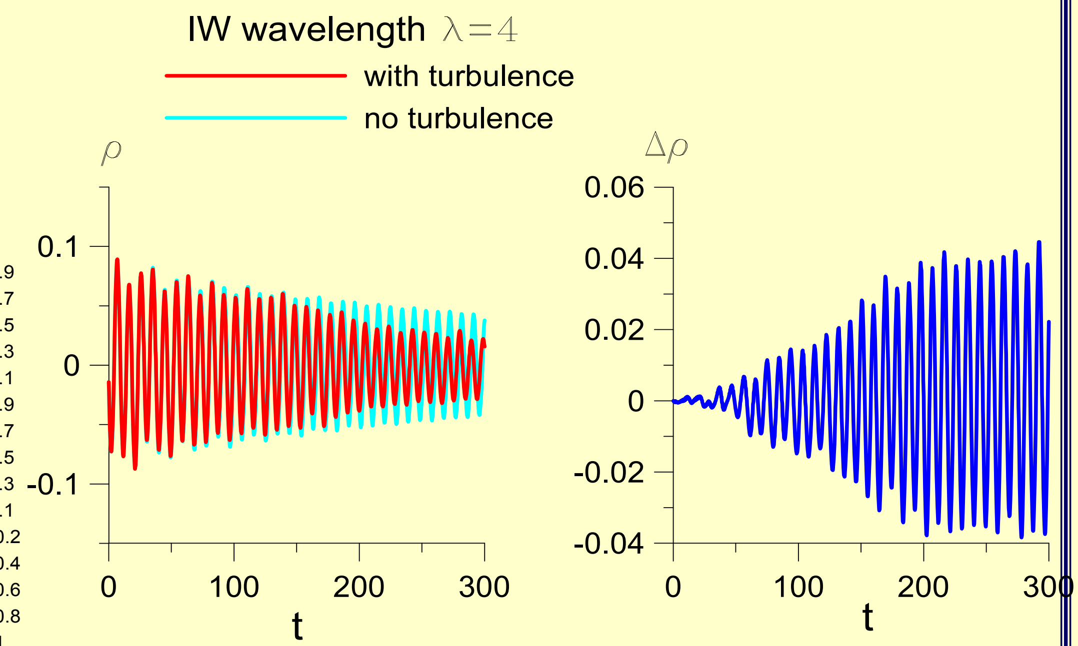


1st IW mode with amplitude 0.1 and wavelength λ = 2π/k=10:



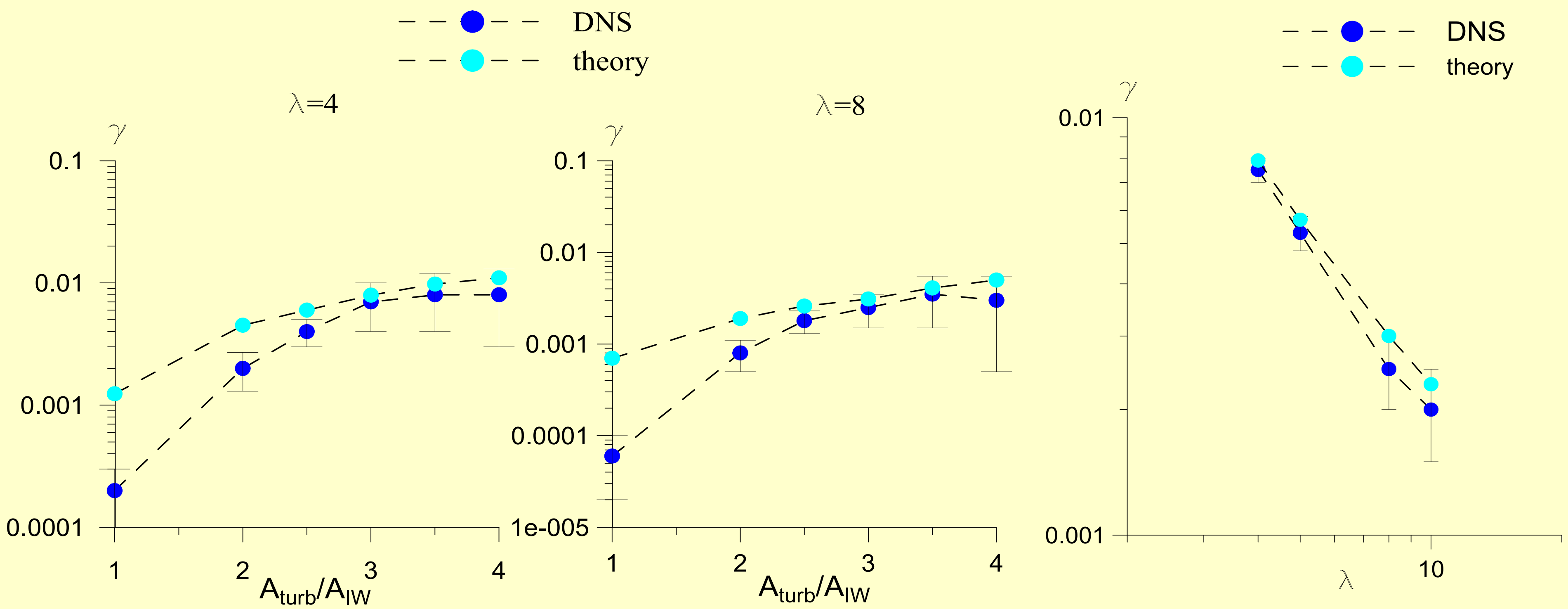
Damping of weak IWs by turbulence





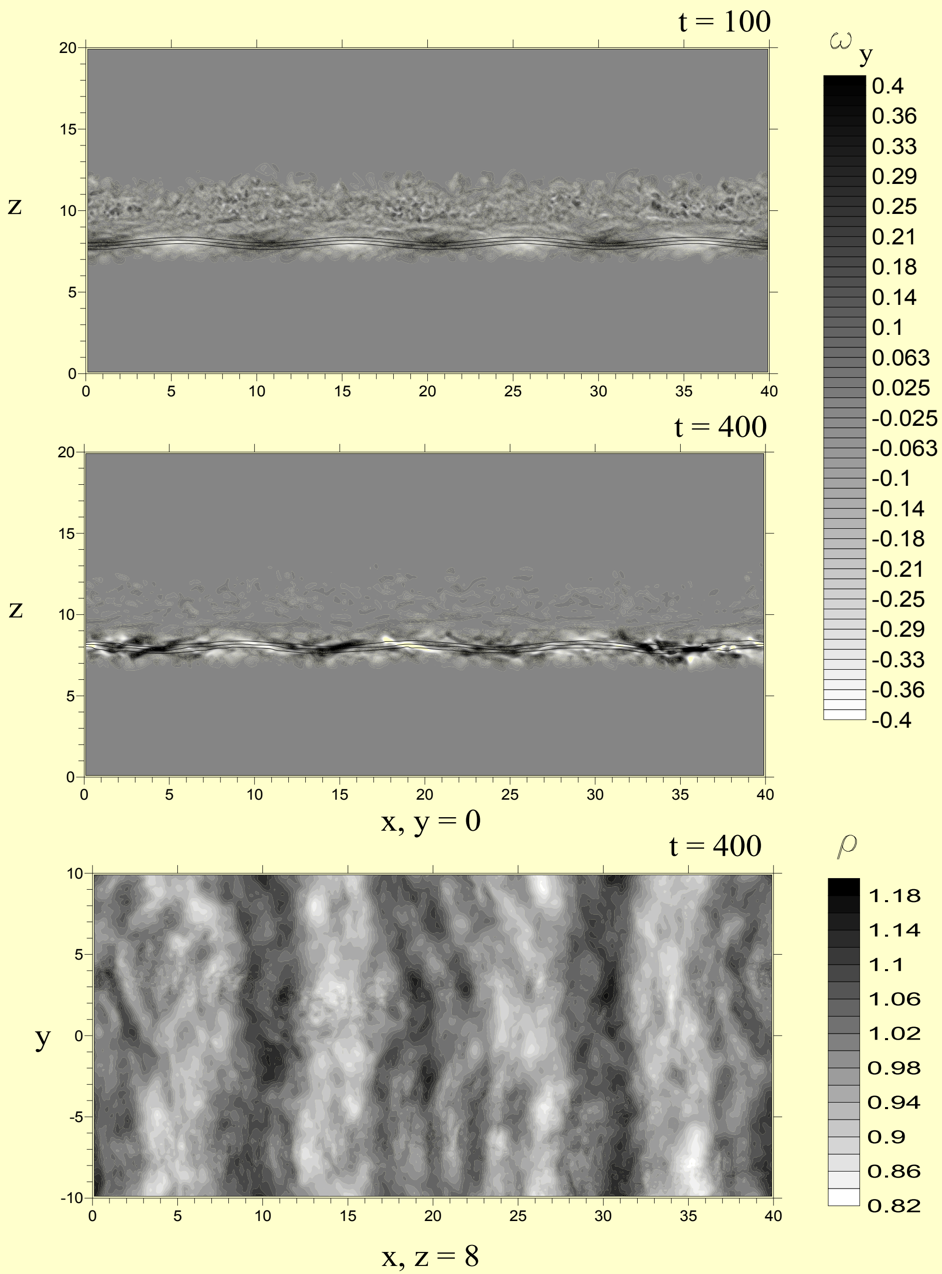
$A_{turb}/A_{IW} = 3$

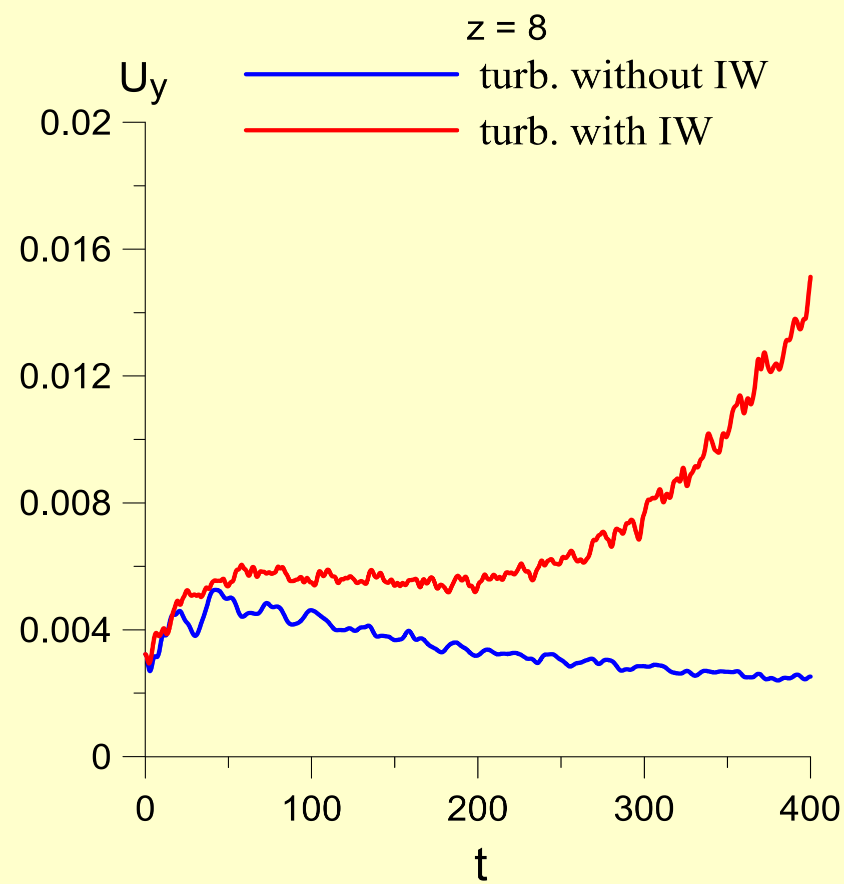
Comparison of DNS results with theory based on RANS (Ostrovsky & Zaborskih 1996)



$A_{turb}/ A_{IW} = 3$

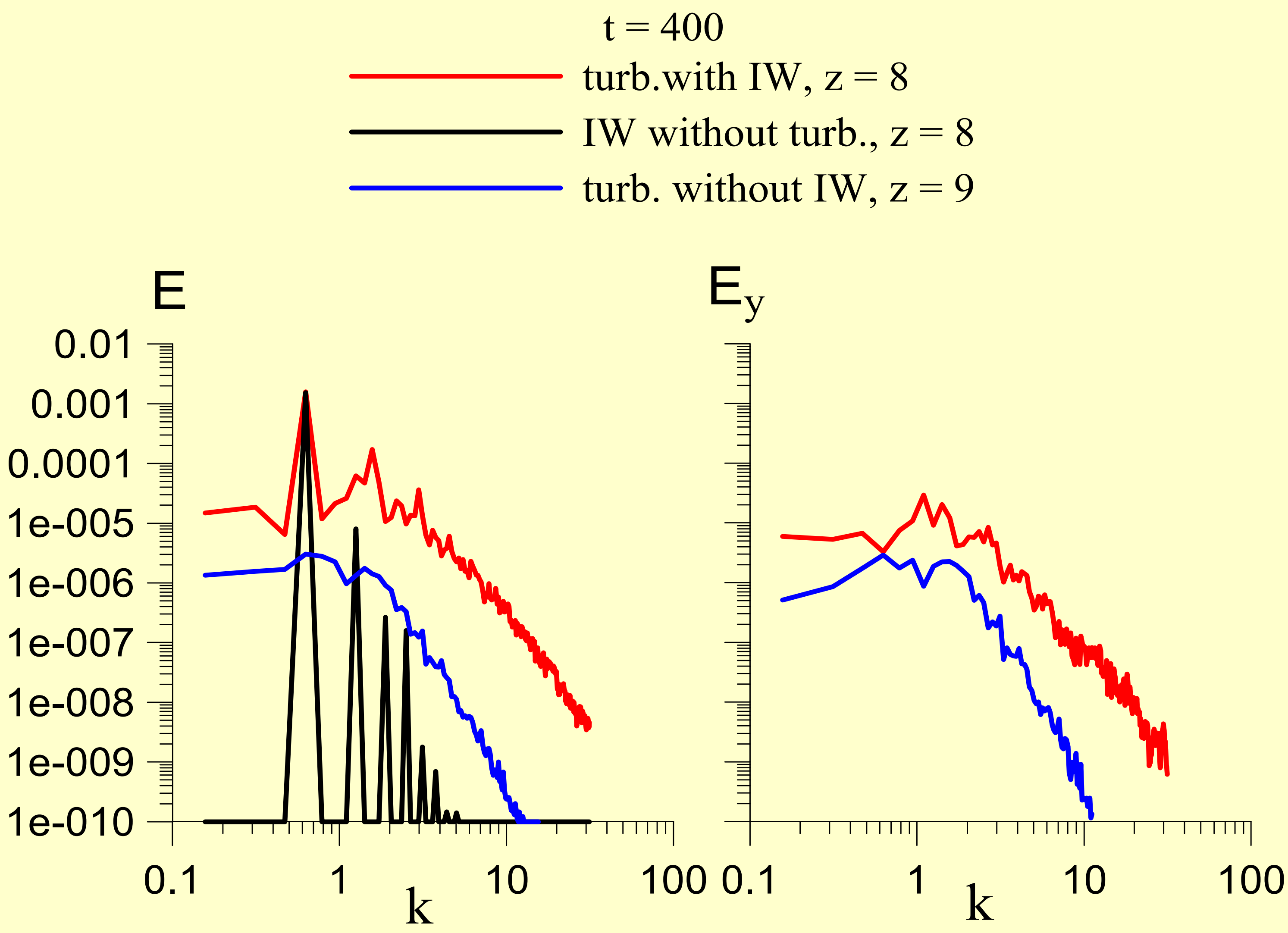
Enhancement of turbulence by strong non-breaking IW





$A_{turb} / A_{IW} = 1$

Enhancement of turbulence by strong non-breaking IW



$A_{turb} / A_{IW} = 1$

CONCLUSIONS

• **DNS are capable of resolving the small-scale mixing processes in the upper ocean and atmospheric boundary layer without introducing *any* modeling typical of RANS and LES with scaling of laboratory and , in some cases, even field experiments.**

• **Stably-stratified atmospheric boundary layer can be significantly modified by the surface waves: DNS predicts both modification of a stationary, weakly stratified turbulent BL, and a pre-turbulent regime supported by the surface waves *even* under very stable stratification;**

• **IWs propagating in the seasonal oceanic pycnocline can effectively interact with small-scale turbulence: *weak* IWs are *damped* by turbulence, whereas *strong* IWs *enhance* turbulence;**

• ***Tasks of ongoing research*: increase resolution of DNS and approach field-experiment parameters; develope coupled models for air-ocean simulations; increase complexity of the interfaces (wave breaking, capillary ripples); take into account gas bubbles in water and water droplets (sea spray) in the air.**