Solution of Financial Mathematics Real-Time Problems by Virtual Supercomputer

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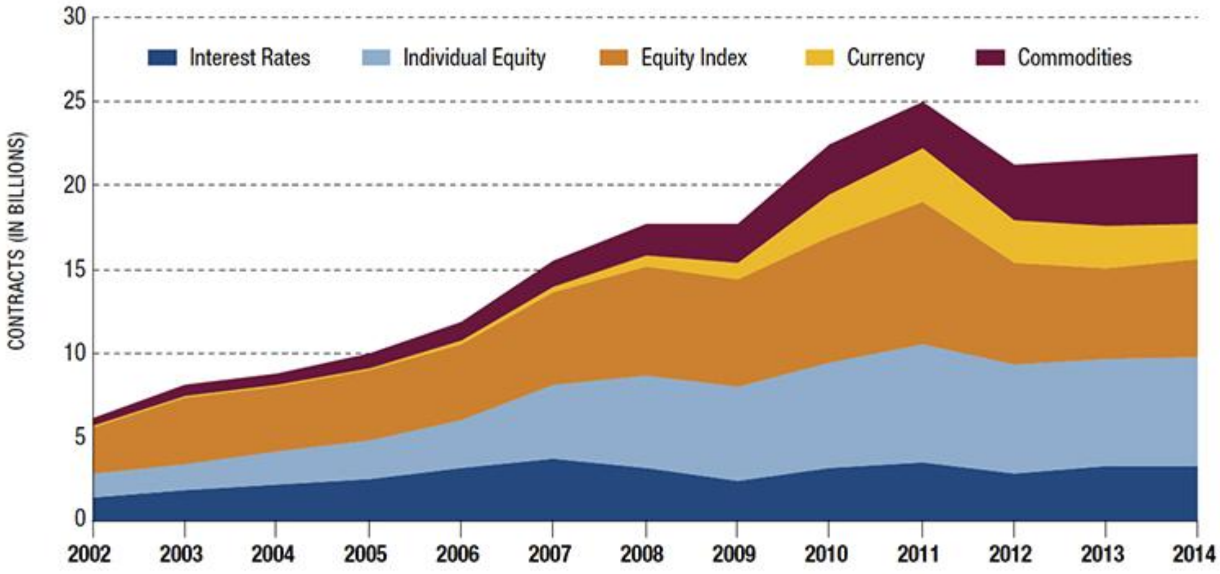
Prerequisites

The rapid growth in derivatives trading
Financial crises
The big risk in derivatives trading
High performance requirements calculation

Prerequisites

The increase in trading volume in the derivatives market



Speed of computing

The speed of decision-making is a key factor in the world of finance, where time goes by at a fraction of a second.
For example, Hibernia Atlantic has decided to connect Europe and the US special communication cable to reduce the transmission of information to 6 milliseconds.

Precision of calculations

The dynamics of S & P500 index on 24 and 25 August: fluctuations reach 1.5 - 2%.

Efficiency of calculations

Main conclusions:

- the main factor is the speed of the calculations;
- enough to accurately to predict the relative accuracy of the market should be 1% or higher.

The mathematical aspect

From the point of view of mathematics for maximum efficiency calculations the following main areas of research:

- correctness of the mathematical problem;
- analysis of the effectiveness of different approaches to solving the problem;
- the impact of input parameters on the solution of the problem.

Main tasks

- A study of existing models to describe the dynamics of change in the option price
- Building a mathematically correct formulation of the problem of pricing an option
- Construction of an algorithm for solving the problem in order to achieve maximum speed calculations
- Creating a software package that implements the proposed algorithms

Task settings

As the basis of the studies the Asian option was taken with the arithmetic average.

The base model is the Black-Scholes model.

The original mathematical model

The main problems of our model:

- the boundary conditions are given only on the part of the area under consideration;
- the boundary conditions are not correlated with each other at some points of the area under consideration.

The modified model

A modified model of the Asian option

$$\frac{\partial C}{\partial t} = \frac{\sigma^2}{2} S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} + S\frac{\partial C}{\partial A} - rC,\ rC,$$

$$C(S,A,0) = \max\left(\frac{A}{T} - K\right),$$

$$C(0,A,\tau) = e^{-r\tau}\max\left(\frac{A}{T} - K, 0\right),$$

$$C(S,A_{max},\tau) = e^{-r\tau}\max\left(\frac{A_{max}}{T} - K, 0\right) + \frac{S}{rT}(1-e^{-r\tau}),$$

$$C(S_{max},A,\tau) = e^{-r\tau}\max\left(\frac{A}{T} - K, 0\right) + \frac{S_{max}}{rT}(1-e^{-r\tau}),$$

$$C(S,0,\tau) = \frac{S}{rT}(1-e^{-r\tau})$$

Approaches

For computing the price of the Asian option the following numerical approaches were used :

- Finite difference method for the solution of the Black – Scholes PDE;
- Monte Carlo method
  - for the stochastic differential equation modeling the dynamics of changes in the price of the underlying asset;
  - for the calculation of the path integral;
  - to calculate the differential equation in partial derivatives;

The numerical tests

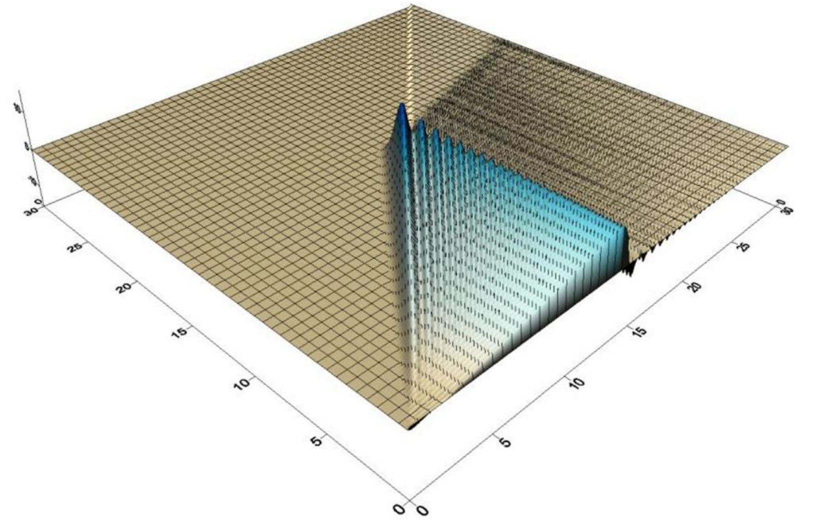
In order to study the problem of mathematical addition of the proposed modified model were considered other options for boundary conditions. In particular, interesting results were obtained for the following combination:

$$C(0,A,t)=0,\quad C(S_{max},A,t)=1,$$

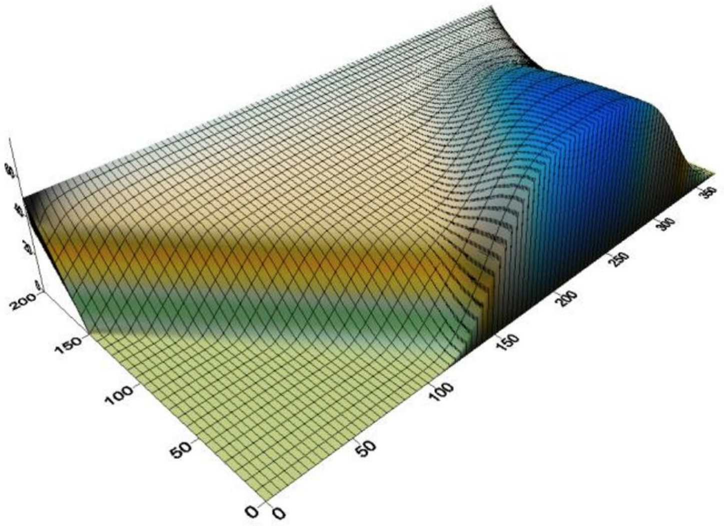
$$C(S,0,t)=1,\quad C(S,A_{max},t)=0.$$

The approbation

Finite-difference method



Zero boundary conditions



Modified boundary conditions

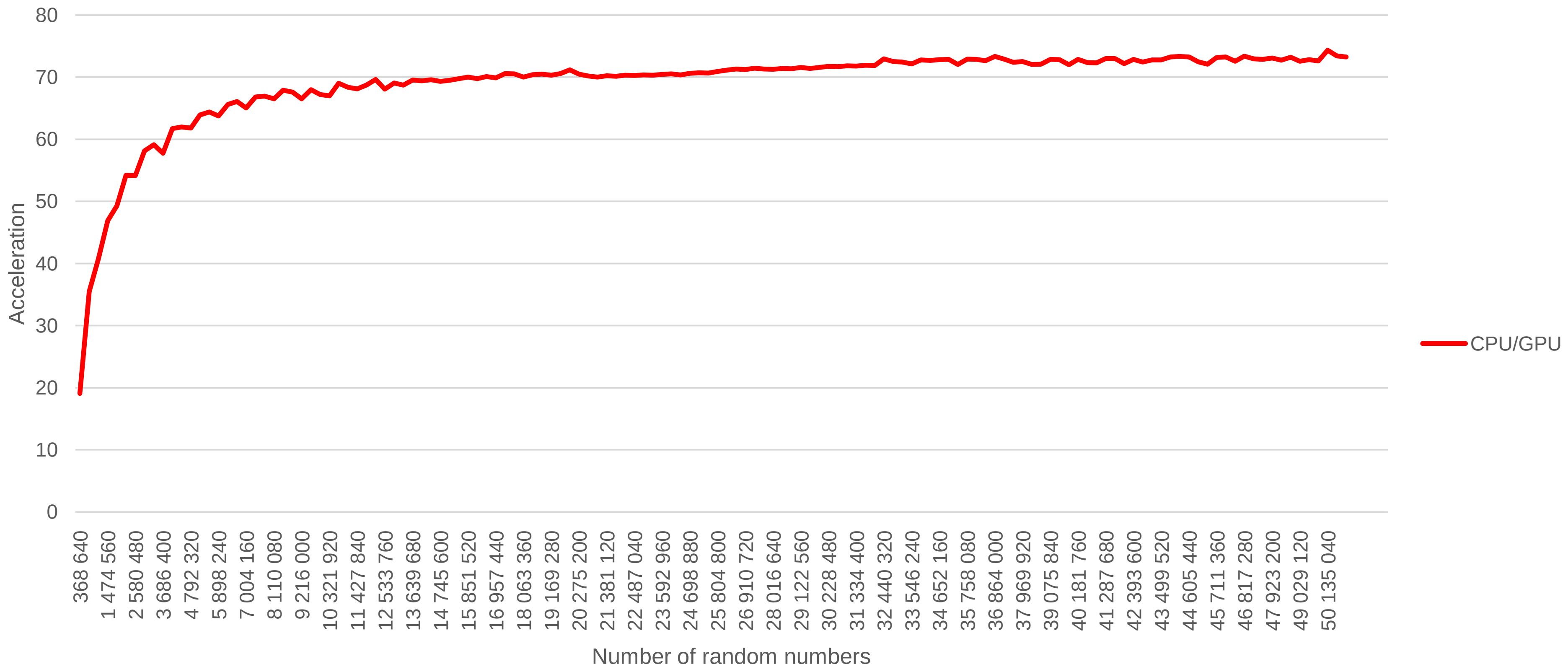
Comparison of approaches

speed is the most effective for approach based on the integration of stochastic differential equation, but it is inferior in accuracy;
path integral method is universal, but it requires a lot of computing resources;
Monte-Carlo method is most efficient to calculate a large number of different types of options;
for the problems of small dimensions and for the calculation of a large number of similar options the most efficient is finite-difference method.

Acceleration of GPGPU computing

Modern GPGPU have a large volume of internal memory, as well as a huge number of nuclei that produces amazing acceleration compared to the CPU. Tasks, using the Monte Carlo method, is particularly well decide to GPGPU.

Acceleration of GPGPU computing

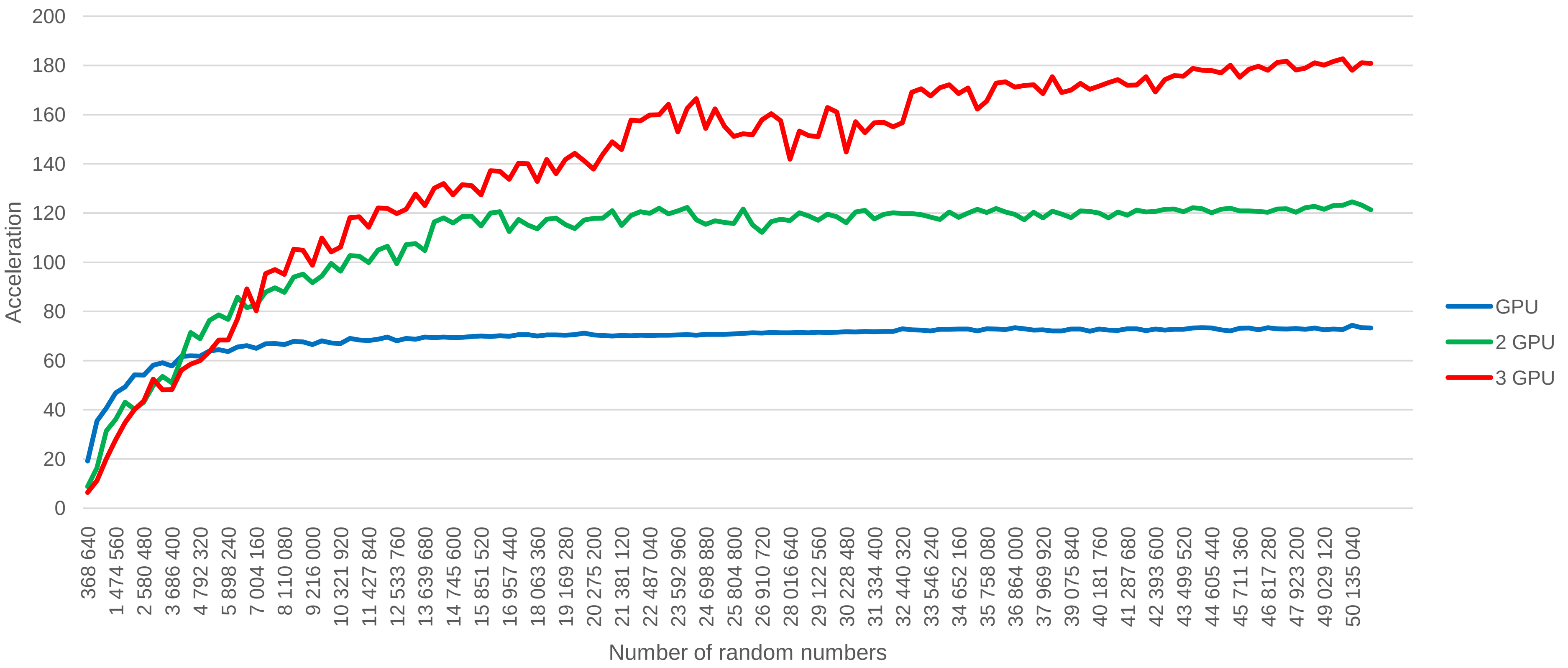


Scaling GPGPU

Experiments on the scalability problem evaluating options for several GPU.

Number of GPGPU	The acceleration
1	1.0x
2	1.6x
3	2.5x

Scaling GPGPU



The main results

1. The correct mathematical model for calculatingof the price of Asian option based on the Black-Scholes model was built.
2. The cases of boundary conditions when the Black-Scholes model is not applicable for computing was detected.
3. The comparative characteristics of different approaches to solving this problem were described.
4. An algorithm for calculating European and Asian options for different initial parameters of the problem was proposed.

The main results

5. Compiled software package that implements the following numerical approaches:
   a) finite difference method for solving the equations of the Black - Scholes partial;
   b) use the path integral to find the price of the European option;
   c) Monte-Carlo:
      i. solving stochastic differential equations
      ii. the calculation of the path integral

THANK YOU FOR ATTENTION!