Implicit scheme for the Maxwell equations solution in case of flat 3D domains

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A standard scheme for Maxwell equation solution in plasma physics is the leap-frog scheme. The advantages of the scheme are second order of accuracy, time reversibility, simplicity, however the stability condition is $c\tau/h<1$, where $h=min\{h_x; h_y; h_z\}$, c – the light speed. In case of flat ultrarelativistic beams in supercolliders the beam sizes ratio $\sigma_x:\sigma_y:\sigma_z$ may be 1:200:60000. The high ratio of vertical beam sizes provides higher luminosity, and the purpose of the experiments is to increase the maximum available luminosity. The high relativistic factor requires the domain boundaries to be close to the beam. The three-dimensional problem of ultrarelativistic beam dynamics in supercolliders is complicated, and the present effective parallel load-balanced algorithms based on the leap-frog scheme limit the beam sizes ratio to ~ 1:50:500. However, the stability condition in case of such flat domains (but not the accuracy condition) forces to decrease the time-step with decreasing of the minimal spatial step, and to decrease the number of times-steps in order to achieve a specified time moment.

We present a new finite-difference scheme for Maxwell's equations solution in three dimensional cases for the cases of different scales in different directions. We overcome the conditional stability by modifying the standard scheme into implicit one. The new scheme satisfies the Gauss law for the electric and magnetic fields in the final-differences, what is important for the whole algorithm of the beam dynamics.

The approximation order and the stability of the scheme in one-dimensional and three-dimensional cases are analyzed. The study of the wave propagation in different directions demonstrated that the both schemes are non-invariant on the angle with the coordinate axes, but the differences between the two schemes results are insignificant. It is shown, that the scheme maintains the amplitude of the wave and maintains the wave propagation speed with the second order in space and time.