New Compact Equation for Numerical Simulation of 1D and 2D Freak-Waves on Deep Water

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We applied canonical transformation to water wave equation not only to remove cubic nonlinear terms but to simplify drastically fourth order terms in Hamiltonian. Unlike in [1],[2] we choose four-wave interaction coefficient in very suitable form:

$$\begin{aligned} \tilde{T}^{kk_1}_{k_2k_3} &= \frac{(kk_1k_2k_3)^{\frac{1}{2}}}{2\pi} min(k,k_1,k_2,k_3)\theta(kk_1k_2k_3) = \\ &= \frac{(kk_1k_2k_3)^{\frac{1}{2}}}{8\pi}\left(k+k_1+k_2+k_3-|k-k_2|-|k-k_3|-|k_1-k_2|-|k_1-k_3|\right)\theta(kk_1k_2k_3) \end{aligned} \tag{1}$$

Equation of motion is the following:

$$\frac{\partial c(x,t)}{\partial t} + i\hat{\omega}_k c(x,t) - i\hat{P}^+\frac{\partial}{\partial x}\left(|c(x,t)|^2\frac{\partial c(x,t)}{\partial x}\right) = \hat{P}^+\frac{\partial}{\partial x}\Big(U(x,t)c(x,t)\Big) \tag{2}$$

here $U(x,t)=\hat{k}|c(x,t)|^2$- advection velocity, $\hat{k}$ is the modulus wavenumber operator and $\hat{P}^+$ is projection operator to the upper half-plane.

The new compact equation (2) generalized for the "almost" 2-D waves i.e. waves slightly inhomogeneous in the transverse direction y. In this case frequency $\hat{\omega}$ depends on both k_x and k_y as $\hat{\omega}_{k_x,k_y}$, while nonlinear terms not depend on y, and c now depends on both x and y:

$$\frac{\partial c(x,y,t)}{\partial t} + i\hat{\omega}_{k_x,k_y} c(x,y,t) - i\hat{P}^+_x\frac{\partial}{\partial x}\left(|c(x,y,t)|^2\frac{\partial c(x,y,t)}{\partial x}\right) = \hat{P}^+_x\frac{\partial}{\partial x}\left(U(x,y,t)c(x,y,t)\right) \tag{3}$$

Due to specific structure of nonlinearity the equation (3) can be effectively solved on the computer.

We have performed numerical simulations of sea surface waving in the framework of equations (2, 3). Initial condition in numerical experiments was chosen as slightly perturbed monochromatic wave. After some time we observed the freak wave formation.

References

[1] Dyachenko, A.I. and Zakharov, V.E.: Compact equation for gravity waves on deep water, JETP Letters, **93(12)**, 701–705 (2011)

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